Portfolio

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Purpose

Throughout this course, you will work through a few prompts and collect them together in a course portfolio. These prompts represent key topics in this course. As you progress through the course, you will add to this portfolio and also have an opportunity to go back and edit any previous responses. By the end of the term, you will have a portfolio of work that demonstrates your understanding of this course.

Assignment

This portfolio will be worth 100 points throughout the term. Each prompt will be given at the beginning of the course, but you will not be able to complete them until you progress through certain topics in the course. For any individual prompt, you can submit it for corrections and critiques, and you will receive feedback to assist you in editing your response.

Your response to each prompt should adhere to these guidelines:

- Each prompt will begin on a new page.
- The instructions for the prompt are written before any work is shown.
- Any computations are fully worked out without the aid of a calculator or computer.
- Any work that is unclear is justified with full sentences.
- The conclusion is written as the end of the response.
- Any values are given exactly, unless rounding is specifically asked for.
- Proper notation is always given.

Prompts

Prompt 1. Consider the linear system below.

$$\begin{cases} 2x - 2y - 4z = -12 \\ x \qquad\quad - \ z = 1 \\ x + \ y + 2z = 10 \end{cases}$$

Solve the system using Gauss-Jordan elimination. Show all row reduction steps, making it clear which elementary row operation you are using.

Prompt 2. Consider the linear system below.

$$\begin{cases} x - 7y \qquad\quad + 6\omega = 5 \\ \qquad\qquad\quad z - 2\omega = -3 \\ -x + 7y - 4z + 2\omega = 7 \end{cases}$$

(a) Write the system of equations as a vector equation.

(b) Write the system as a matrix equation $A\mathbf{x} = \mathbf{b}$.

(c) Solve the system of equations using linear algebra techniques. Indicate your solution in parametric vector form.

Prompt 3. Let $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ -8 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ -11 \\ -6 \end{bmatrix}$.

(a) Is $\mathbf{b} \in \text{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$?

(b) Is $\{\mathbf{u}.\mathbf{v}, \mathbf{w}\}$ linearly independent or linearly dependent? If the set is linearly dependent, write a linear dependence relation.

Prompt 4. Find the standard matrix A for the linear transform $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that T first reflects points through the horizontal x_1-axis, secondly rotates points by an angle of $\frac{-\pi}{3}$ about the origin, and lastly reflects points through the line $x_2 = x_1$.

Prompt 5. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. Find A^{-1}. Do not use Cramer's Method or the adjugate/adjoint method to find A^{-1}.

Prompt 6. Let $A = \begin{bmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 3 & 9 & -12 & 7 & 1 & 15 \\ -3 & -9 & 13 & -10 & 6 & -15 \\ 1 & 3 & -3 & -1 & 6 & 6 \end{bmatrix}$.

(a) What does it mean for a vector $\mathbf{x}$ to be in $\operatorname{Col} A$?

(b) What does it mean for a vector $\mathbf{x}$ to be in $\operatorname{Nul} A$?

(c) Find a basis for $\operatorname{Col} A$.

(d) Find a basis for $\operatorname{Nul} A$.

(e) Find a basis for $\operatorname{Row} A$.

(f) Find a basis for $\operatorname{LNul} A$.

(g) Find $\dim \operatorname{Col} A$, $\dim \operatorname{Nul} A$, $\dim \operatorname{Col} A^T$, $\dim \operatorname{Nul} A^T$.

(h) Find $\operatorname{rank} A$.

Prompt 7. Let $A \in M_{3\times 3}$ and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ as defined below.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \quad , \quad \mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \quad , \text{ and } \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}.$$

(a) Show that the characteristic polynomial for A is $-\lambda^3 + \lambda^2 + 4\lambda - 4$.

(b) Find the eigenvalues of A (Hint: Use factor-by-grouping).

(c) Find a basis for the eigenspace of each eigenvalue.

(d) Diagonalize A – that is, find two matrices, D and P, such that $A = PDP^{-1}$. The matrix D must be a diagonal matrix.

(e) Find $A\mathbf{u}$, $A\mathbf{v}$, and $A\mathbf{w}$ without using a matrix-vector product.

Prompt 8. Let $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ as defined below.

$$\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \quad , \text{ and } \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}.$$

(a) Find $\mathbf{u} \cdot \mathbf{w}$.

(b) Determine if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal or not. Show any work to support your conclusion.

(c) Normalize $\mathbf{u}$.