Steps For Solving Equations Containing Rational Expressions

1. Write out the original problem.
2. Prep each fraction.
3. Find LCD on the side, by factoring denominators and stacking like factors.
4. Check all factors containing a variable in the *LCD* for **<u>restrictions</u>**. Restrictions are found by setting these factors $\neq$ **0** and solving. Write the restrictions by the *LCD*. Go back to each fraction and write each denominator in its factored form and put each numerator in the ( ).
5. Multiply each side of the equation by the *LCD* by putting each side of equation in [ ] and put the *LCD* over (1) in front of each [ ]
6. Continue the multiplication process by distributing the *LCD* times each individual fraction within the [ ]. After distributing the *LCD* to each fraction, keep each fraction in a [ ] and put the sign of each fraction outside of the [ ]/
7. Cancel out the like factors in the numerator and denominator and you will end up with just the numerators.
8. **CAUTION: It is important to be careful here to notice when you are solving for a variable in an equation and when you are just adding and subtracting fractions. DO NOT multiply through by the *LCD* unless you have an equation and are <u>solving</u>.**
9. After all of the denominators have been eliminated, simplify each side of the equation.
10. If the highest degree of the variable term is to the 1^{st} power, solve the equation by getting the variable term on the left and the constant term on the right.
11. If the highest degree is over 1 then get all terms on left side and solve by factoring or the quadratic formula.
12. Check each answer by first making sure it is not a restriction and then substitute if a formal check is needed. Write down all solutions after checking.

Example:

Solve for x: $\dfrac{4}{4x^2 - 9} - \dfrac{5}{4x^2 - 8x + 3} = \dfrac{8}{4x^2 + 4x - 3}$

Steps on next page.

Notes on Solving:	Simplification
Write out problem and notice that this is an equation and that the instructions did state to solve.	$\frac{4}{4x^2-9}-\frac{5}{4x^2-8x+3}=\frac{8}{4x^2+4x-3}$
(On the side get *LCD*).	$4x^2-9=(2x+3)(2x-3)$ $4x^2-8x+3=(2x-3)(2x-1)$ $4x^2+4x-3=(2x+3)(2x-1)$ $LCD=(2x+3)(2x-3)(2x-1)$
(On the side find the restrictions using the factors in the *LCD*.)	$2x+3\neq 0 \text{ or } 2x-3\neq 0 \text{ or } 2x-1\neq 0$ $2x\neq -3 \text{ or } 2x\neq 3 \text{ or } 2x\neq 1$ $\frac{2x}{2}\neq -\frac{3}{2} \text{ or } \frac{2x}{2}\neq \frac{3}{2} \text{ or } \frac{2x}{2}\neq \frac{1}{2}$ $x\neq -\frac{3}{2} \text{ or } x\neq \frac{3}{2} \text{ or } x\neq \frac{1}{2}$ **Restrictions:** $x\neq -\frac{3}{2} \text{ or } x\neq \frac{3}{2} \text{ or } x\neq \frac{1}{2}$
Rewrite each fraction with a factored denominator and each numerator in ( ).	$\frac{(4)}{(2x+3)(2x-3)}-\frac{(5)}{(2x-3)(2x-1)}=\frac{(8)}{(2x+3)(2x-1)}$
Put each side of the fraction in a [ ] and then multiply each side by the *LCD*.	$(2x+3)(2x-3)(2x-1)\left[\frac{(4)}{(2x+3)(2x-3)}-\frac{(5)}{(2x-3)(2x-1)}\right]=(2x+3)(2x-3)(2x-1)\left[\frac{(8)}{(2x+3)(2x-1)}\right]$
Distribute the *LCD* to each fraction in [ ], keep a [ ] around each fraction. Cancel like factors.	$\left[\frac{\cancel{(2x+3)}\cancel{(2x-3)}(2x-1)(4)}{\cancel{(2x+3)}\cancel{(2x-3)}}\right]-\left[\frac{(2x+3)\cancel{(2x-3)}\cancel{(2x-1)}(5)}{\cancel{(2x-3)}\cancel{(2x-1)}}\right]=\left[\frac{\cancel{(2x+3)}(2x-3)\cancel{(2x-1)}(8)}{\cancel{(2x+3)}\cancel{(2x-1)}}\right]$
All denominators cancel, continue to solve equation and this takes many steps.	$[(2x-1)(4)]-[(2x+3)(5)]=[(2x-3)(8)]$ $[8x-4]-[10x+15]=[16x-24]$ $8x-4-10x-15=16x-24$ $8x-10x-4-15=16x-24$
The equation is a 1st order equation so we will work on getting the variable terms on the left and the constant terms on the right.	$-2x-19=16x-24$ $-2x-19+19=16x-24+19$ $-2x=16x-5$ $-2x-16x=16x-16x-5$ $-18x=-5$ $\frac{-18x}{-18}=\frac{-5}{-18}$
The answer is $x=\frac{5}{18}$ and this is not a restriction so it is a valid solution if it checks.	$x=\frac{5}{18}$

Note: Check if required.

We now will check the answer by substituting $x = \frac{5}{18}$ in to the original equation. Note, it is very involved.

Check $\frac{4}{4x^2 - 9} - \frac{5}{4x^2 - 8x + 3} = \frac{8}{4x^2 + 4x - 3}$ ***for*** $x = \frac{5}{18}$

$$\frac{4}{4\left(\frac{5}{18}\right)^2 - 9} - \frac{5}{4\left(\frac{5}{18}\right)^2 - 8\left(\frac{5}{18}\right) + 3} = \frac{8}{4\left(\frac{5}{18}\right)^2 + 4\left(\frac{5}{18}\right) - 3}$$

$$\frac{4}{4\left(\frac{25}{324}\right) - 9} - \frac{5}{4\left(\frac{25}{324}\right) - 8\left(\frac{5}{18}\right) + 3} = \frac{8}{4\left(\frac{25}{324}\right) + 4\left(\frac{5}{18}\right) - 3}$$

$$\frac{4}{\frac{100}{324} - 9} - \frac{5}{\frac{100}{324} - \frac{40}{18} + 3} = \frac{8}{\frac{100}{324} + \frac{20}{18} - 3}$$

$$\frac{4}{\frac{25}{81} - 9} - \frac{5}{\frac{25}{81} - \frac{20}{9} + 3} = \frac{8}{\frac{25}{81} + \frac{10}{9} - 3}$$

$$\frac{4}{\frac{25}{81} - \frac{9}{1}} - \frac{5}{\frac{25}{81} - \frac{20}{9} + \frac{3}{1}} = \frac{8}{\frac{25}{81} + \frac{10}{9} - \frac{3}{1}}$$

$$\frac{4}{\frac{25}{81} - \frac{9(81)}{1(81)}} - \frac{5}{\frac{25}{81} - \frac{20(9)}{9(9)} + \frac{3(81)}{1(81)}} = \frac{8}{\frac{25}{81} + \frac{10(9)}{9(9)} - \frac{3(81)}{1(81)}}$$

$$\frac{4}{\frac{25}{81} - \frac{729}{81}} - \frac{5}{\frac{25}{81} - \frac{180}{81} + \frac{243}{81}} = \frac{8}{\frac{25}{81} + \frac{90}{81} - \frac{243}{81}}$$

$$\frac{4}{\frac{-704}{81}} - \frac{5}{\frac{88}{81}} = \frac{8}{\frac{-128}{81}}$$

$$4\left(\frac{-81}{704}\right) - 5\left(\frac{81}{88}\right) = 8\left(\frac{-81}{128}\right)$$

$$\frac{-81}{176} - \frac{405}{88} = -\frac{81}{16}$$

$$\frac{-81}{176} - \frac{405(2)}{88(2)} = -\frac{81}{16}$$

$$\frac{-81}{176} - \frac{810}{176} = -\frac{81}{16}$$

$$-\frac{891}{176} = -\frac{81}{16}$$

$$-\frac{81}{16} = -\frac{81}{16} \checkmark$$